CHAPTER 3

AN ANALYTICAL APPROACH TO ESTIMATE JITTER

In this chapter an analytical approach to estimate jitter is given. Section 3.1 deals with the analytical derivation of jitter for the CBR traffic when multiplexed with Poisson traffic. This is basically a review of the work done in [RG92]. Section 3.2 shows how this derivation can be modified to calculate jitter when the CBR traffic is multiplexed with self-similar background traffic. The proofs for all the expressions derived in this section are given in Appendix A.

3.1 Jitter in ATM networks handling Poisson traffic

In [RG92], jitter is considered in a discrete time process where the time-unit is arbitrary ie they consider a multiplexer having a service time of 1 cell/ time slot. As shown in Fig 3.1 a CBR (periodic) stream is considered which has a inter-cell interval of 'd' time slots. The i^{th} cell has a sojourn time in the system(multiplex or network) of $D+W_i$, where D is a constant (propagation time, etc) and W_i is a non-negative delay component introduced by the multiplexer (waiting time in the muliplexer queue) . Without loss of generality D is assumed to be 0 for the rest of this section(D depends only on the route followed by the cells of the connection considered).

W_i is assumed to constitute a stationary ergodic process with a probability distribution:

$$w_k=\Pr\{W_i=k\} \quad \text{for } k \geq 0 \qquad ...(3.1)$$

It is further assumed that the dependence between successive delays is first-order Markovian (ie delay of the i^{th} cell depends only on the delay of the $(i-1)^{th}$ cell), characterised by the transition probabilities:

$$q_{jk}=\Pr\{W_{i=k} / W_{i-1}=j\} \quad \text{for } j,k\geq 0 \qquad \text{...(3.2)}$$

Then w_k satisfies the equation:

$$w_k = \sum_j w_j q_{jk} \qquad \text{...(3.3)}$$

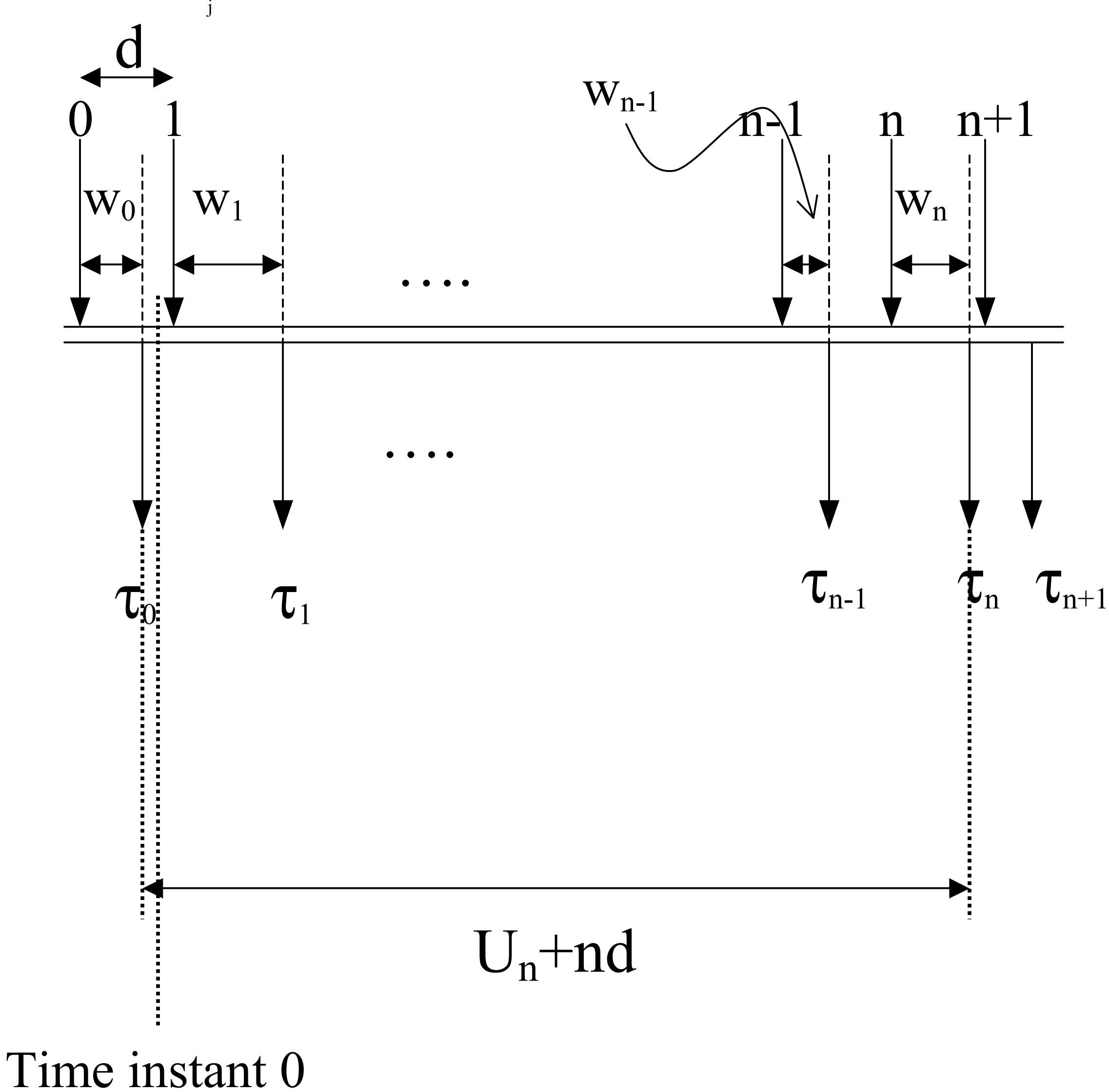

Fig 3.1 The Multiplexer Model

Inter -cell exit time (delay) distributions

Let 0 be an arbitrary time instant and let τ_0 be the cell exit instant immediately preceding 0. Let τ_i ,$i \geq 1$, be the exit instants of subsequent cells. Define the random variable U_n :

$$U_n = \tau_n - \tau_0 - nd \quad \Rightarrow \tau_n - \tau_0 = U_n + nd \qquad \text{...(3.4)}$$

U_n is the variation of the n^{th} order inter-exit time w.r.t the inter-arrival interval 'nd'. Jitter is characterised by the distributions of the random variable U_n, $n \geq 1$, and especially that of $U_1 = W_1 - W_0$. The distribution of U_1 allows comparisons between the inter-arrival in the jittered process with that in the initial flow that is constant and exactly equal to d.

Let $f_n(k)$ be the distribution of U_n ie,:

$$f_n(k) = \Pr\{U_n = k\} \qquad \text{...(3.5)}$$

It can be proved that

$$f_n(k) = \sum_{i \geq 0} w_i q^{(n)}_{i,i+k} \qquad \text{...(3.6)}$$

where, $q^{(n)}_{i,j}$ is the i-j component of the nth power of the transition matrix, $q_{jk,}$.

Jitter due to a multiplexing stage

It is assumed that the multiplex receives the superposition of a periodic stream of cells of period 'd' (CBR stream) and a Poisson stream of rate λ. It is assumed that the multiplex can only start to transmit cells at specific instants....,-2,-1,0,1,2,..., and the periodic cells arrive just before a service instant (this implies that 'd' is an integer). With these assumptions, the queue at the moment of a periodic arrival is a Markov process.

Assuming FIFO service, the queue length at the i^{th} periodic cell arrival is identical to the waiting time W_i introduced earlier.

To calculate the transition probabilities q_{ij} we introduce the conditional probabilities:

$$Q(j,k) = \Pr\{W_i > k / W_{i-1} = j\} \qquad ...(3.7)$$

$$P_n(j,k) = \Pr\{W_i > k / W_{i-1} = j \text{ and n Poisson arrivals in } ((i-1)d, id)\} \qquad ...(3.8)$$

We then have from the definitions of q_{jk} and Q(j,k-1)

$$q_{jk} = Q(j,k-1) - Q(j,k), \qquad ...(3.9)$$

It can be shown that

$$Q(j,k) = \sum_{n} P_n(j,k) \frac{(\lambda d)^n}{n!} e^{-\lambda d} \qquad ...(3.10)$$

It can also be proved that

$$P_n(j,k) = \begin{cases} 0 & \text{for } (j+1 \geq d \text{ and } n \leq d+k-j-1) \text{ or } (j+1 < d \text{ and } n \leq k) \\ \sum_{s=1}^{n-k} \binom{n}{s+k} \left(\frac{s}{d}\right)^{s+k} \left(1-\frac{s}{d}\right)^{n-s-k} \frac{d-n+k}{d-s} & \text{for } j+1 < d \text{ and } k < n \leq d+k-j-1 \\ 1 & \text{for } n > d+k-j-1 \end{cases}$$

...(3.11)

The delay distribution w_k is deduced on solving the state equations (3.3). To solve these equations, first $P_n(j,k)$ is calculated using (3.11) . Using this is in (3.10) Q(j,k) is obtained . q_{jk} is determined from (3.9) using Q(j,k) calculated above. w_k is then obtained using (3.3) . Next $f_1(k)$ is calculated using (3.6). $f_1(k)$ gives the probability that the first cell is delayed with respect to the zeroeth cell by 'k' slots. Since there is no

absolute time reference it can be inferred that $f_1(k)$ gives the probability that the n^{th} cell is delayed with respect to the n-1^{th} cell by 'k'. We now find that value of k which gives us 99% of the area under the $f_1(k)$ curve. The significance of this value of k is that it gives us the value of the buffer size required so that the n^{th} cell is not lost more than 99% of the time when compared to the n-1^{th} cell. Since this true for all n, the value of k gives the buffer size required so that the CLR does not exceed 1%.

When done for Poisson background traffic with a λ=0.75 and the CBR stream's period d=30 the buffer size required was found to be 11 slots.

3.2 Jitter in ATM networks handling Self-Similar traffic

In order to analyse jitter for the self-similar case, we need the distribution function of the self-similar traffic. We propose a distribution function that models self-similar traffic by capturing its heavy-tailed behaviour. Heavy-tailed distributions are generally used to describe traffic processes such as packet inter-arrival times and burst lengths. The distribution of a random variable X is said to be heavy-tailed if

$\Pr[X>x] \sim x^{-\alpha} \quad , \quad \alpha>0$

One common heavy tailed function used to model active periods of 'on-off' sources is the Pareto distribution [PKC] with the density function

$$f(x) = \frac{\alpha}{k}\left(\frac{k}{x}\right)^{\alpha+1} \qquad \text{where } x>k \text{ and } \alpha>0 \qquad ...(3.12)$$

The Hurst parameter in this case is given by $H=(3-\alpha)/2$

The Pareto process cannot be used to model the arrival process as it is a continuous distribution. Hence we looked at a discretised version of the Pareto process as suggested in [TG97] :

$$\Pr\{N=n\}=\frac{n^{-(\alpha+1)}}{\sum_{1}^{\infty} n^{-(\alpha+1)}} \qquad ,\alpha>0 \qquad ...(3.13)$$

The mean of this process is given by

$$E[N]=\sum_{1}^{\infty} n \frac{n^{-(\alpha+1)}}{\sum_{1}^{\infty} n^{-(\alpha+1)}} \qquad ...(3.14)$$

The infinite summations in the above expressions were evaluated using the approximation:

$$\sum_{1}^{\infty} n^{-(\alpha+1)} = \int_{1}^{\infty} \mathrm{x}^{-(\alpha+1)} \mathrm{dx} \qquad ...(3.15)$$

Hence (3.14) reduced to

$E[N]=\alpha/(\alpha-1)$ ...(3.16)

We see that if we fix the mean , α gets fixed and hence the process gets fixed ie this becomes a single parameter process, the parameter being the mean. Such a density function cannot be used to model a self-similar arrival processes as we need to be able to simulate various traffic traces with different means for the same Hurst parameter. For the existing self-similarity models like the FGN , the mean, variance and the Hurst parameter are independent of each other ie, it is a 3 parameter process. But a density function does not exist for this model, it is only defined by means of its autocorrelation.

We propose a modified version of the discrete Pareto process: a 2 parameter, discrete Pareto Process defined by

Pr{N(t)=n}=Probability of n arrivals in time t

$$=\frac{(\mathrm{nk}+1/\mathrm{t})^{-(\alpha+1)}}{\sum_{1}^{\infty}(\mathrm{nk}+1/\mathrm{t})^{-(\alpha+1)}} \qquad ...(3.17)$$

The parameter α is a function of the Hurst parameter.The constant k helps us to get desired means for a fixed value of H.The three parameters for this process is the H and the mean. The mean of this process was found using the approximation in (3.15).

$$E[N(t)] = \frac{tk\alpha + 1}{tk(\alpha - 1)} \qquad ...(3.18)$$

The motivation for us to add the 1/t term in (3.17) stemmed from two reasons. Firstly, In (3.13) n is not allowed to take the value 0. But zero arrivals should be possible in an arrival process. Hence we needed to add a term to n in the density function to make it possible for n=0. Secondly, it is obvious that the mean no. of arrivals in a time t should increase with an increase in t We see from (3.18) that as t increases E[N(t)] increases. Had we added t instead of 1/t , then the mean would have decreased with an increase in d. We also see that by fixing α the mean does not get fixed ie, for the same value of α , by varying k we can obtain different means. This was the reason for including k in (3.17). We created a random no. generator using the distribution function given in (3.17) and found that α = 1.5 yielded a Hurst parameter of around 0.8.

The exact relationship between α and H was'nt found.

To use this in the analytical method described in Sec.(3.1), only one change is made. In (3.10), the Poisson distribution function is replaced by the distribution function of the 2-Parameter discrete Pareto Process ie ,

$$Q(j,k) = \sum_{n} P_n\,(j,k)\,\frac{(nk + 1/d)^{-(\alpha+1)}}{\sum_{1}^{\infty}(nk + 1/d)^{-(\alpha+1)}} \qquad ...(3.19)$$

One important point to be noted here is that in deriving (3.11), we make use of the fact that Poisson arrivals are uniformly distributed in any finite interval. It is not wrong to make the same assumption for self-similar traffic because as given in [weN] self-similarity depends only the no. of arrivals in a given time interval and not on the internal distribution of the arrivals. The authors of [weN] have reported to have got the same H parameter for traces in which they shuffled all the arrivals in a time interval maintaining the no. of arrivals in the time interval.

As described in Sec3.1, the buffer length was calculated in this case ie, Self-similar background traffic described by (3.17) with $H=0.8$ ($\alpha=1.5$), $d= 0$ and a mean of 22.5 (to make the utilisation 75%) for a CLR of less than 1%.

The buffer length was found to be 70 as against a buffer length of 11 for Poisson traffic.