## **CHAPTER 4**

## SYSTEM SIMULATION

In order to do the jitter analysis, self-similar traffic traces must be generated. This was done using the algorithm proposed in [VP97]. This algorithm will be referred to as the Fast FFT method in the rest of the paper. Section 4.1 deals with this algorithm. An ATM multiplexer, carrying self-similar background traffic and the periodic stream of interest (CBR traffic) was then simulated and the cell delay variation was obtained. This is described in Section 4.2.

## 4.1 GENERATION OF SELF-SIMILAR TRAFFIC USING FAST FFT METHOD

The principle behind this method can be summarized as below.

Suppose the power spectrum  $f(\lambda;H)$ , of the FGN signal is known, a sequence of complex number  $z_i$  can be constructed corresponding to this power spectrum.  $z_i$  is in a sense a frequency domain sample path. An inverse DTFT is then used to obtain the time-domain counterpart  $x_i$ . Because  $x_i$  has (by construction) the power spectrum of FGN, and because autocorrelation and power spectrum form a Fourier pair,  $x_i$  is guaranteed to have the autocorrelational properties of an FGN process (which for many purposes are its most salient characteristics). The algorithm is explained below. For an FGN process the power spectrum is [B86]:

$$f(\lambda;H) = A(\lambda;H)[|\lambda|^{-2H-1} + B(\lambda;H)] \quad \text{for } 0 < H < 1 \text{ and } -\pi < \lambda < \pi \qquad \dots (4.1)$$

where :

$$A(\lambda;H) = 2\sin(\pi H)\Gamma(2H+1)(1-\cos\lambda) \qquad \dots (4.2)$$

(1.0)

$$B(\lambda; H) = \sum_{j=1}^{\infty} [(2\pi j + \lambda)^{-2H-1} + (2\pi j - \lambda)^{-2H-1}] \qquad \dots (4.3)$$

The infinite summation in (4.3) can be approximated as

$$B(\lambda; H) \approx a_1^d + b_1^d + a_2^d + b_2^d + a_3^d + b_3^d + \frac{a_3^{d'} + b_3^{d'} + a_4^{d'} + b_4^{d'}}{8H\pi} \qquad \dots (4.4)$$

Where

$$d = -2H-1$$
  

$$d' = -2H$$
  

$$a_k = 2k\pi + \lambda$$
  

$$b_k = 2k\pi - \lambda$$
  
...(4.5)

Define  $f(\lambda;H)$  as the approximation of (4.1) using (4.4). The inputs are H and n(desired ,even no. of observations). The algorithm proceeds as follows:

- 1. Construct a sequence of values  $\{f_1, \dots, f_{n/2}\}$ , where  $f_j = f(2\pi j/n; H)$ , corresponding to the power spectrum of an FGN process for frequencies from  $2\pi/n$  to  $\pi$ .
- "Fuzz" each {f<sub>i</sub>} by multiplying it by an independent exponential random variable with mean 1. Call the fuzzed sequence {f''<sub>i</sub>}. ...(4.6)
- 3. Construct  $\{z_1,...,z_{n/2}\}$ , a sequence of complex values such  $|z_i| = \operatorname{sqrt}(f''_i)$  phase of  $z_i$  is uniformly distributed between 0 and  $2\pi$ .
- 4. Construct  $\{z'_1,...,z'_{n-1}\}$ , and "expanded" version of  $\{z_1,...,z_{n/2}\}$ :

$$z'_{j} = \begin{cases} 0, & \text{if } j = 0 \\ \frac{z_{i},}{z_{n-j}}, & \text{if } 0 < j \le n/2 \\ \frac{z_{n-j}}{z_{n-j}}, & \text{if } n/2 < j < n \end{cases}$$

where  $\overline{z_{n-j}}$  denotes the complex conjugate of  $z_{n-j}$ .  $\{z'_j\}$  retains the power spectrum used in constructing  $\{z_i\}$ , but it is symmetric about  $z'_{n/2}$ , it now corresponds to the Fourier transform of a real-valued signal.

 Inverse Fourier transform of {z'<sub>j</sub>} is taken to obtain the approximate FGN sample path {x<sub>i</sub>}.

Appendix B gives the source code in Matlab for implementing the above method.

FGN has mean zero. This implies the presence of negative samples in  $\{x_i\}$ . But in order to make use of the samples to simulate no. of packet arrivals per timeslot, the mean is shifted by adding to each point in the sample the absolute value of min $\{x_i\}$ .

This gives us an FGN trace with a mean of  $\min\{x_i\}$ . In order to get a desired mean for the trace, the entire trace is scaled by a constant. In our simulations we used the period of the CBR traffic ('d') as 30 slots and hence to make the utilization 0.75, we scaled our FGN trace to a mean of 22.5. Since the no. of packets arriving per slot is an integer we rounded each sample in our trace to the nearest integer.

This is only an approximate method for generating self-similar traces. The author of [VP97] only argues that this method only *effectively* produces FGN. Since we did not obtain the H parameter that was input to the algorithm (verified using VT plot), we used the H as given by the VT plot.

## **4.2 SIMULATION OF JITTER IN AN ATM MULTIPLEXER**

To simulate jitter in an ATM network, the multiplexer model shown in Fig. was used. For the purpose of simulation, a 'd' of 30 was assigned. In between two CBR packets, self-similar arrivals were introduced with a mean of 22.5, i.e. in between two CBR packets we introduce one sample from the trace obtained in Sec 4.1. As stated in [WE 9], the internal distribution of the arrival of packets per time interval considered is not important for the LRD, which only depends on the no. of arrivals per time interval. Hence, we assumed the self-similar packets arrived uniformly in 'd' slots.

The delay was calculated for each of the CBR packets using the following algorithm:

 The buffer occupancy is checked just before the arrival of a CBR packet using the following formula Buffer occupancy B = the no. of self-similar arrivals occurring in the previous 'd' slots+the delay of the previous CBR packet - d ....(4.7)

- 2. If B<0 the delay for the CBR packet is 0(as the multiplexer buffer becomes empty before the arrival of the CBR packet).
- 3. If B>0, the delay for the CBR packet is B.

The above algorithm was also carried out for Poissonian arrivals (with the same utilization of 0.75, i.e. mean =22.5) between two CBR cells. The Poisson arrivals are uniformly distributed over a given time period (in this case, 'd' time slots). The Matlab source code for the above simulations are given in Appendix

The calculations in both the self-similar and the Poisson cases were done for 32768 CBR packets.

The histogram of the delays of the CBR packet was calculated and normalized to get the PDF of the delays.

The buffer requirement (in terms of time slots) was determined from the PDF in order to have CLR of not more than 1%. The buffer length required is the value of the delay at which the area under the PDF curve becomes 99%.

For the Poisson process the buffer length required for a CLR less than 1% was found to be 6 (for 75% utilization and 'd'=30 slots)

Table 4.1 gives the Buffer length required for a CLR less than 1% when the CBR traffic stream is multiplexed with self-similar background traffic for various values of H. Also shown in the table is the ratio of self-similar buffer size requirement and the Poisson buffer requirement. The figures that follow Table 4.1 graphically the cell delay distribution for various cases.

| Hurst Parameter(H) | Buffer length required for | Ratio of buffer requirement |
|--------------------|----------------------------|-----------------------------|
|                    | CLR < 1%                   | of self-similar and Poisson |
|                    | (self-similar case)        | traffic                     |
| 0.53               | 7                          | 1.167                       |
| 0.59               | 10                         | 1.667                       |
| 0.6                | 11                         | 1.833                       |
| 0.67               | 14                         | 2.33                        |
| 0.71               | 21                         | 3.50                        |
| 0.75               | 37                         | 6.16                        |
| 0.8                | 58                         | 9.66                        |
| 0.85               | 51                         | 8.5                         |
| 0.86               | 97                         | 16.16                       |
| 0.9                | 226                        | 37.67                       |
|                    |                            |                             |

Table 4.1 Buffer length requirements and the Ratio of Buffer length requirements as a function of the Hurst parameter.



figure1: Delays of CBR packets when multiplexed with self-similar traffic of H=0.7 and utilisation 0.75 figure2: Delays of CBR packets when multiplexed with Poisson traffic with utilisation 0.75









blue-self similar background traffic with H=0.6 , buffer length required for CLR<1%=11 red - poisson background traffic, buffer length required for CLR<1%=6 utilization=0.75 in both cases



PROBABILITY OF DELAY AS A FUNCTION OF DELAY (for the CBR packets)

blue-self similar background traffic with H=0.7 , buffer length required for CLR<1%=21 red - poisson background traffic, buffer length required for CLR<1%=6 utilization=0.75 in both cases



PROBABILITY OF DELAY AS A FUNCTION OF DELAY (for the CBR packets) blue-self similar background traffic with H=0.8 ,buffer length required for CLR<1%=58 red - poisson background traffic, buffer length required for CLR<1%=6 utilization=0.75 in both cases



PROBABILITY OF DELAY AS A FUNCTION OF DELAY (for the CBR packets)

blue-self similar background traffic with H=0.9 , buffer length required for CLR<1%=226 red - poisson background traffic, buffer length required for CLR<1%=6 utilization=0.75 in both cases